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# Main Metric Invariants of Finite Metric Spaces. III

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**Abstract**—We find new main metric invariants of finite metric spaces. All these invariants are allotted to three sets.

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We continue investigations started at [1–3]. Consider a set $\mathbb{K}$ of finite metric spaces of the same cardinality $N > 1$. Our goal is to construct three sets whose union contains all the known main metric invariants defined on the set $\mathbb{K}$. We present new basic metric invariants for $N > 3$ in each of the three sets. The new proofs are more optimal, and the results are more general.

As already noted in [1–3], the main invariants can be applied in classification of finite metric spaces, their recognition, and also in selection of finite metric space classes with given properties.

**Definition 1** ([3]). The function $F : \mathbb{K} \to \mathbb{R}_+$ taking values in the set of all nonnegative real numbers $\mathbb{R}_+$, is a *main metric invariant* (on $\mathbb{K}$), if the following conditions hold:

(i) $F(X) = F(Y)$ for any pair $X, Y \in \mathbb{K}$ of isometric metric spaces;

(ii)

$$F(X) \in \{\rho(x, y) : x, y \in X,\ x \neq y\}$$

for any metric space $(X, \rho) \in \mathbb{K}$;

(iii)

$$|F(X) - F(Y)| \leq 2d_s(X, Y),$$

for arbitrary metric spaces $(X, \rho), (Y, d) \in \mathbb{K}$, here [1]

$$d_s(X, Y) = \frac{1}{2}\min\{\operatorname{dis} f : f : X \to Y \text{ is a bijection}\},$$

$$\operatorname{dis} f = \max\{|\rho(x, y) - d(f(x), f(y))| : x, y \in X\}.$$

We introduce three new definitions.

**Definition 2.** Consider $S \subset X$, $\operatorname{card}(S) = k$, here $2 \leq k \leq N$, $\mathrm{Ld}(S)$ is the set of all distances between different points of $S$ written in non-decreasing order. $\mathrm{Ld}_n(S)$ is an $n$th element of $\mathrm{Ld}(S)$, here $1 \leq n \leq C_k^2$. $\mathrm{Ld}_{mnk}(X)$ is an $m$th element of the nondecreasingly ordered list of elements $\mathrm{Ld}_n(S)$, obtained by searching all subsets $S \subset X$ of the fixed power $k$, here $1 \leq m \leq C_N^k$. Note that

$$\mathrm{Ld}_{m12}(X) = \mathrm{Ld}_{1mN}(X) = \mathrm{Ld}_m(X), \quad \mathrm{Ld}_{11k}(X) = \mathrm{Ld}_1(X) = \mathrm{R}_{N-1}(X),$$

$$\mathrm{Ld}_{21k}(X) = \mathrm{Ld}_2(X) = \mathrm{R}_{N-2}(X), \quad \mathrm{Ld}_{C_N^k, C_k^2, k}(X) = \mathrm{Ld}_{C_N^2}(X) = \mathrm{D}(X).$$

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